Space-Time Turbo Equalization Scheme with Beam Steering Based on SVD and QRD over Frequency Selective Fading MIMO Channels

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Abstract
To support broadband multiple-input multiple-output (MIMO) systems using a single carrier transmission scheme under frequency selective fading conditions, this paper proposes a new space-time turbo equalization scheme that employs beam steering technique based on singular value decomposition (SVD) and QR decomposition (QRD) are employed to suppress mutual interference between eigen-beams called inter-beam interference (IBI). In addition, soft interference cancellation followed by minimum mean square error filtering (SC/MMSE) type turbo equalizer is applied to each eigen-beam signal sequence to suppress co-beam interference (CBI) defined as inter-symbol interference (ISI) on the co-eigen-beam, as well as the residual IBI. Computer simulation confirms that the proposed scheme is effective in improving spectral efficiency without extra bandwidth with low transmit power.

I. INTRODUCTION

The rapid progress in broadband wireless communications has led to an increased demand for more efficient and sophisticated signal processing techniques, because available radio spectrum is limited, and it can not satisfy demands for broadband services without a significant increase in communication spectral efficiency. As its key solution, utilizing space-domain signal processing in multiple-input multiple-output (MIMO) channels allows us to achieve higher spectral efficiency without requiring extra radio spectrum. In general, the space-domain processing can be classified into the receiver processing type (e.g., V-BLAST), transmitter processing type, and receiver/transmitter processing type (e.g., SVD-Based). In MIMO channels, singular value decomposition (SVD)-based systems are convenient because eigen-beams formed by beam steering on both ends enable them to be free from inter-beam interference (IBI), and actually, they can provide better performance than the other types of space domain processing because beam steering is employed on both ends [1]. On the other hand, QR decomposition (QRD)-based V-BLAST schemes that employ beam steering only on the receiver side are suitable for creating independent subchannels if beam steering on the transmitter side is unavailable, because beam steering even on the receiver side can suppress inter-channel interference (ICI) caused by the spatial multiplexing, although it is not completely free from ICI [2], [3].

In addition to this IBI or ICI mitigation, we also have to cope with severe frequency selective fading caused by inter-symbol interference (ISI) to support high bit rate transmission services. At present, there are many literatures that propose OFDM-based MIMO to improve ISI immunity. However, employing OFDM in the MIMO systems causes large increase in peak-to-average power ratio (PAPR), which is not preferable from the viewpoint of transmission power efficiency. Therefore, the problem of the high PAPR brings us to a concept based on single carrier transmission employing turbo equalization techniques which are capable of mitigating ISI by low-complexity iterative receiver manner [4], [5]. Ref. [4]'s turbo equalizer (soft interference cancellation followed by minimum mean square error filtering: SC/MMSE) consists of a soft interference canceller and a linear adaptive filter based on the MMSE criterion. To reduce more computational complexity on the matrix inversion to determine taps of adaptive filter, applying a matched filter approximation has been proposed in [6]. In addition, iterative MIMO equalization process applied to SC/MMSE schemes have been proposed in [7].

In this paper, to support broadband MIMO systems using a single carrier transmission scheme with lower-complexity of the hardware under frequency selective fading conditions, we will propose a new space-time turbo equalization scheme that employs beam steering technique based on SVD and QRD on both ends to suppress unwanted multipath components included in each eigen-beam. Specifically, after the strongest path in the multipath components is selected, SVD is applied to it to create eigen-beams to be used in the data transmission. At this stage, although eigen-beams for the selected path is kept orthogonal to each other, there is IBI from different multipath components. Thus, to suppress a part of such IBI, QRD is applied to the non-selected path components in the beam steering on the receiver because beam steering on the transmitter is invalid for them. Then, SC/MMSE type turbo equalizer is applied to each eigen-beam signal sequence to suppress co-beam interference (CBI) defined as ISI on the co-eigen-beam, as well as the residual IBI. With this scheme the computational complexity on matrix inversion involved in finding log-likelihood ratio (LLR) can be greatly reduced in the turbo equalizing process.

Performances of the proposed scheme are evaluated by computer simulation. The results show that the proposed scheme is effective in the reduction of transmit power with higher spectral efficiency. In addition, they also demonstrate effectiveness of QRD.
II. SYSTEM DESCRIPTION

Fig. 1 shows the configuration of transmitter and receiver expressed in the equivalent lowpass system. The channel model considered here consists of $M$ transmit elements and $N$ receive elements in an array, respectively. In the following, it is assumed for simplicity that $M \geq N$. At the first, the main stream of information bits $\{d\}$ is divided into $I$ sub-streams $\{d_i\}$ through serial to parallel (S/P) converter where $I$ corresponds to number of eigen-beams constituted by the beam steering on the both transmitter and receiver sides. The substreams $\{d_i\}$ are then encoded, interleaved, and mapped onto symbol to derive modulated symbols. After multiplying the modulated symbols $a_i(k)$ by the antenna weights described as matrix $\tilde{V}(l_\lambda)$ in order to form the transmitted symbols $s_m(k)$, the signal is transmitted over frequency selective MIMO channels, where $k$ indicates index of the transmitted symbol timing and meaning of $\tilde{V}(l_\lambda)$ will be explained later.

At the receiver, discrete time measurement at the $n$-th antenna yields the sampled value series $r_n(k)$ of the antenna output as

$$r_n(k) = \sum_{l=0}^{L-1} \sum_{m=1}^{M} h_{nm}(l) s_m(k-l) + \nu_n(k)$$  \hspace{1cm} (1)

where $L$ and $l$ denote the channel memory length and the index of multipath components, respectively. On top of that, $\nu_n(k)$ denotes complex additive white Gaussian noise (AWGN) term at the $n$-th receive antenna, where noise at each timing is uncorrelated and its variance is $\sigma^2$. The complex gain of each multipath component $h_{nm}(l)$ are assumed to be time-invariant in a frame and known to the transmitter and the receiver.

We define here the following vectors and matrices:

$$r(k) = [r_1(k), r_2(k), \ldots, r_N(k)]^T,$$

$$H(l) = \begin{bmatrix} h_{1,1}(l) & \ldots & h_{1,M}(l) \\ \vdots & \ddots & \vdots \\ h_{N,1}(l) & \ldots & h_{N,M}(l) \end{bmatrix},$$

$$s(k) = [s_1(k), s_2(k), \ldots, s_M(k)]^T,$$

and

$$\nu(k) = [\nu_1(k), \nu_2(k), \ldots, \nu_N(k)]^T,$$

where $H(l)$ denotes MIMO channel matrix for each $l$-th path.

Then, Eq. (1) can be expressed in matrix form as

$$r(k) = \sum_{l=0}^{L-1} H(l) s(k-l) + \nu(k).$$  \hspace{1cm} (6)

A. Beam steering and Space-Time Sampling

The antenna weights matrix on the transmitter is calculated by SVD of $H(l)$ with the multipath components having the largest first eigenvalue. SVD is well known that it is an appropriate way of diagonalizing matrices and it can be written as

$$H(l_\lambda) = U(l_\lambda) \cdot D(l_\lambda) \cdot V^H(l_\lambda)$$  \hspace{1cm} (7)

where $l_\lambda$ is the index of a path with the largest first eigenvalue, $D(l_\lambda)$ is a diagonal matrix of real, nonnegative singular value, and $V^H$ signifies conjugate transpose (Hermitian). Note that, eigenvalues defined as the square of singular values in the diagonal matrix $D(l_\lambda)$ are descending order, i.e., the first eigenbeam has the largest eigenvalue. Based on SVD, multiplying the modulated symbols by $V(l_\lambda)$ and the received symbols by $V^H(l_\lambda)$ allows the channel matrix $H(l_\lambda)$ to diagonalize.

In addition, the maximum number of available eigen-beams is corresponded with $\text{rank}[D(l_\lambda)] = N$. In the case of $I < N$, $V(l_\lambda)$ reduced to $\tilde{V}(l_\lambda)$ defined as $M \times I$ matrix extracted up to $I$-th column from $V(l_\lambda)$. In short, $\tilde{V}(l_\lambda)$ is applied to antenna weights matrix in order to eliminate the IBI in the $l_\lambda$-th path components. Eq. (6) can be rewritten as

$$r(k) = \sum_{l=0}^{I-1} H(l) \tilde{V}(l_\lambda) a(k-l) + \nu(k)$$  \hspace{1cm} (8)

where

$$a(k) = [a_1(k), a_2(k), \ldots, a_I(k)]^T$$  \hspace{1cm} (9)

and

$$\tilde{V}(l_\lambda) = \begin{bmatrix} v_{1,1}(l_\lambda) & \ldots & v_{1,I}(l_\lambda) \\ \vdots & \ddots & \vdots \\ v_{M,1}(l_\lambda) & \ldots & v_{M,I}(l_\lambda) \end{bmatrix}.$$  \hspace{1cm} (10)

On top of that, we define the following vectors and matrices to express the received signal $r(k)$ in space-time domain representation:

$$r(k) = [r^T(k+L-1), r^T(k+L-2), \ldots, r^T(k)]^T$$  \hspace{1cm} (11)

and

$$H = \begin{bmatrix} H(0) & \ldots & H(L-1) & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & H(0) & \ldots & H(L-1) \end{bmatrix},$$  \hspace{1cm} (12)

$$\tilde{V}(l_\lambda) = \begin{bmatrix} \tilde{V}(l_\lambda) & 0 \\ \vdots & \ddots \\ 0 & \tilde{V}(l_\lambda) \end{bmatrix},$$  \hspace{1cm} (13)

$$a(k) = [a(k+L-1), \ldots, a(k), \ldots, a(k-L+1)]^T,$$  \hspace{1cm} (14)

and

$$\nu(k) = [\nu^T(k+L-1), \ldots, \nu^T(k)]^T.$$  \hspace{1cm} (15)

where $\cdot$ signifies space-time domain.
Finally, Eq. (8) is expressed in space domain as

$$\mathbf{r}(k) = \mathbf{H} \mathbf{\bar{V}}(l_\lambda) \mathbf{a}(k) + \mathbf{\eta}(k). \quad (16)$$

Fig. 2 shows configuration of the SC/MMSE equalizer with the proposed beam steering scheme. The equalizer is comprised of a beam steering, soft interference canceller, MMSE detector, and soft-input soft-output (SISO) channel decoder. At the receiver side, $\mathbf{U}^H(l_\lambda)$ enables the IBI components of the $l_\lambda$-th path in the space time domain received signal $\mathbf{r}(k)$ to be eliminated completely. However, it can not suppress the IBI of the other paths because $\mathbf{U}^H(l_\lambda)$ is calculated from $\mathbf{H}(l_\lambda)$. In other words, a V-BLAST type signal processing on the receiver side is additionally necessary to suppress such IBI coming from the other paths. Therefore, based on the concept of V-BLAST scheme, QRD is applied to suppress IBI components in each path other than $l_\lambda$-th one. Because the first eigen-beam has the largest eigenvalue, it is reasonable to decompose $\mathbf{H}(l_\lambda) \mathbf{V}(l_\lambda)$ into unitary matrix $\mathbf{Q}(l)$ and lower triangular matrix $\mathbf{R}(l)$ in order to free the received signal from IBI at the first eigen-beam. Then, using the estimated symbols, IBI at the other eigen-beams can be suppressed. This QRD process can be expressed as

$$\mathbf{H}(l_\lambda) \mathbf{V}(l_\lambda) = [\mathbf{q}_1(l), \ldots, \mathbf{q}_I(l)] \begin{bmatrix} r_{1,1}(l) & 0 \\ \vdots & \ddots \\ r_{I,1}(l) & \cdots & r_{I,I}(l) \end{bmatrix} = \mathbf{Q}(l) \mathbf{R}(l). \quad (17)$$

Consequently, the weights matrix in space-time domain $\mathbf{\bar{U}}^H(l_\lambda)$ on the receiver side is decided as

$$\mathbf{\bar{U}}^H = \text{diag} \left[ \tilde{\mathbf{Q}}^H(l_\lambda - 1), \ldots, \tilde{\mathbf{Q}}^H(l_\lambda + 1), \tilde{\mathbf{Q}}^H(l_\lambda), \mathbf{R}(l) \right] \quad (18)$$

where $\tilde{\mathbf{Q}}(l)$ and $\tilde{\mathbf{U}}(l_\lambda)$ denote $N \times I$ matrix extracted up to I-th column from $\mathbf{Q}(l)$ and $\mathbf{U}(l_\lambda)$ respectively. After the beam steering, $\mathbf{r}(k)$ is modified as

$$\mathbf{y}(k) = \mathbf{\bar{U}}^H \mathbf{r}(k) = \mathbf{\bar{U}}^H \mathbf{\bar{V}}(l_\lambda) \mathbf{a}(k) + \mathbf{\eta}(k) = \mathbf{\Psi} \mathbf{a}(k) + \mathbf{\eta}(k) \quad (19)$$

where

$$\mathbf{y}(k) = [\mathbf{y}^T(k + L - 1), \mathbf{y}^T(k + L - 2), \ldots, \mathbf{y}^T(k)]^T. \quad (20)$$

Because $\mathbf{\bar{U}}$ is a unitary matrix, the variance of $\mathbf{\eta}(k)$ is $\sigma^2$, namely there is no noise enhancement.

To extract the component of $a_i(k)$ included in the received symbols, Eq. (22) is, first of all, divided into $I \times I$ square sub-matrices:

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{\Psi}_{1,1} & \cdots & \mathbf{\Psi}_{1,2(L-1)} \\ \vdots & \ddots & \vdots \\ \mathbf{\Psi}_{L,1} & \cdots & \mathbf{\Psi}_{L,2(L-1)} \end{bmatrix} \quad (24)$$

Next, we form $L \times 2(L - 1)$ sliced matrices $\mathbf{\Psi}_i^{ij}$ by extracting components with corresponding index $(i, j)$ in each square sub-matrix where $i = j = 1, \ldots, I$. Then, we define here following sliced vectors:

$$\mathbf{a}_i'(k) = [a_i(k + L - 1), \ldots, a_i(k - 1), a_i(k)]^T \quad (25)$$

and

$$\mathbf{y}_i(k) = [y_i(k + L - 1), y_i(k + L - 2), \ldots, y_i(k)]^T. \quad (26)$$

Finally, the detected $\mathbf{y}_i'(k)$ can be written as

$$\mathbf{y}_i'(k) = \mathbf{\Psi}_i^{ij} \mathbf{a}_i'(k) + \sum_{j=1 \atop j \neq i}^{I} \mathbf{\Psi}_i^{ij} \mathbf{a}_j'(k). \quad (27)$$

In Eq. (27), the first term consists of interested symbol and IBI. It is important to note that the first eigen-beam $(i = 1)$ is completely free from IBI. On the other hand, the second term denotes CBI.

B. Soft Interference Cancellation and Adaptive Filtering

Soft interference canceller requires soft estimated symbols for the modulated symbols in each eigen-beam. The most important part of soft cancellation process is that it is conducted in descending order of eigenvalues, i.e., from $a_1(k)$ to $a_I(k)$ in order every iteration. Therefore, when there are already extrinsic information $\lambda_2(a_j(k))$ at a certain iteration, using them to form soft replicas of IBI and CBI,

$$\tilde{a}_j(k) = \tanh \left( \frac{\lambda_2|a_j(k)|}{2} \right) \quad (j < i) \quad (28)$$

allows improvement of its performance within few iterations. where meaning of $\lambda_2|a_j(k)|$ will be explained later. Otherwise, using the a priori LLR (see Ref. [5]), the detector forms soft estimates of these symbols as

$$\tilde{a}_j(k) = \tanh \left( \frac{\lambda_1|a_j(k)|}{2} \right) \quad (j \geq i). \quad (29)$$
The soft replicas formed by multiplying the soft estimated symbols by \( \Psi'_{i,j} \) are then subtracted from \( y'_i(k) \) to produce the IBI and CBI suppressed \( i \)-th eigen-beam signal vector.

\[
\hat{y}'_i(k) = y'_i(k) - \sum_{j=1}^{I} \Psi'_{i,j} \bar{a}_j(k)
\]

(30)

where

\[
\bar{a}_j(k) = \begin{cases} 
\tilde{a}_i(k + L - 1), \ldots, \tilde{a}_i(k + 1), 0, & (j = i) \\
\tilde{a}_j(k - 1), \ldots, \tilde{a}_j(k - L + 1) & (j \neq i)
\end{cases}
\]

(31)

Next, adaptive linear filter is used to suppress the residual IBI and CBI component: the vector \( w_i(k) \) of the filter taps is determined that the following Mean Square Error (MSE) between the filter output and the signal point corresponding to the detected \( i \)-th eigen-beam is minimized as

\[
w_i(k) = \arg \min_{w_i(k)} \| w_i(k)H \hat{y}'_i(k) - a_i(k) \|^2.
\]

(32)

Since the derivation of the optimum vector \( w_i(k) \) follows Ref. [4], only the results are shown below as

\[
w_i(k) = \left[ \sum_{j=1}^{I} \Psi'_{i,j} A_{i,j} \Psi'^H_{i,j} + \sigma^2 E_L \right]^{-1} \Psi'_{i,i} e_L
\]

(33)

where \( e_L \) is vector of length \( 2L - 1 \) whose elements are all zero except the \( L \)-th element which is one and \( E_L \) is \( L \times L \) identity matrix.

\[
A_{i,j}(k) = \begin{cases} 
E_{2(L-1)} & (j = i) \\
-\text{diag}[\tilde{a}_i^2(k + L - 1), \ldots, \tilde{a}_i^2(k + 1), 0, \tilde{a}_j^2(k - 1), \ldots, \tilde{a}_j^2(k - L + 1)] & (j \neq i).
\end{cases}
\]

(34)

Consequently, the filter outputs are derived as

\[
z_i(k) = w'^H_i(k) \hat{y}_i(k).
\]

(35)

The most important part in this calculation is the matrix inversion, and its computational complexity is \( O(L^3) \) whereas that of conventional SC/MMSE schemes cost \( O(N^3 L^3) \). In addition, applying the matched filter approximation can further reduce the number of computation [6]. Such reduction of the number of computation is especially effective when we would like to increase the number of eigen-beams to be multiplexed because it requires larger number of receive antenna elements.

The extrinsic information to be delivered to the following channel decoder [5] can be derived as

\[
\lambda_0[a_i(k)] = \frac{4M[nz_i(k)]}{1 - \mu(k)}
\]

(36)

where

\[
\mu_i(k) = e^T_L \Psi'^H_{i,i} \left[ \sum_{j=1}^{I} \Psi'_{i,j} A_{i,j} \Psi'^H_{i,j} + \sigma^2 E_L \right]^{-1} \Psi'_{i,i} e_L.
\]

(37)

According to SISO channel decoding iterative process described in Ref. [5], the extrinsic information \( \lambda_1[a_i(k)] \) and decoded bit stream \( \{d\} \) can be derived.

III. NUMERICAL RESULTS

Performances of the proposed system in quasi-static 10-path Rayleigh and rich scattering environments are evaluated by computer simulation. We take the quasi-stationary viewpoint that the channel time variation is negligible over a frame, and that the channel state information is estimated perfect. In addition an exponentially decaying 10-path model with its attenuation of 10 dB between first and tenth path is assumed in this simulation. Tab.1 summarizes the specifications of the simulated system.

A. Bit Error Ratio (BER) performances

Fig. 3 shows BER vs. transmit \( E_b/N_0 \) performances for the proposed system with a parameter of the number of iteration, where \( I = 1 \) (no IBI condition) is assumed. The transmit \( E_b \) is defined as the information bit energy for each eigen-beam on the transmitter. Although they are affected by the CBI, the iterative process gradually reduces the CBI with the increase of iteration. The figure also shows, as a reference performance, the BER curve of the conventional SC/MMSE proposed by Wang for one transmit and four receive antennas case, where the number of iteration is four. Transmit \( E_b/N_0 \) required by the proposed system at the second iteration in BER=10^{-4} is approximately 2 dB smaller than that of the conventional system. This gain is caused by transmit diversity effect by SVD-based beam steering for the path having the largest eigenvalue. Figs. 4 and 5 show BER performances at the fourth iteration for \( I = 2 \) and \( I = 3 \), respectively. Both figures show the case that they are subject to IBI and CBI. It is important to note that errors at the second or third eigen-beam affect the first eigen-beam performance due to residual IBI component. However, the Fig. 4 confirms the iterative process is capable of suppressing their influences to the negligible level. It is important to note that I = 2 transmission is effective in spectral efficiency without extra bandwidth and
transmit power because the BER performance for the second eigenbeam which is worse than the performance for the first eigenbeam is almost the same as that for the conventional (Wang’s) system. On the other hand, Fig. 5 shows the case that the $I = 3$ transmission is subject to serious IBI. As a result, influences of IBI and CBI remains obvious because iterative process cannot completely suppress the IBI and CBI. These results confirms that the $I = 3$ transmission causes 1.5 dB loss of transmit $E_b/N_0$ for the first eigen-beam at the fourth iteration in BER=$10^{-4}$ compared to that of $I = 1$ transmission whereas $I = 2$ transmission does 0.1 dB.

B. Effectivity of QRD

Fig. 6 shows performances in the case employing zero-forcing instead of QRD at the fourth iteration for $I = 3$ in order to confirm the effectivity of the scheme applied QRD. Even though the system with zero-forcing is free from IBI for all eigen-beams, it requires more power compared to the proposed system. These results confirms that the $I = 3$ transmission with zero-forcing causes approximately 2 dB loss of transmit $E_b/N_0$ for the first eigen-beam at the fourth iteration in BER=$10^{-4}$ and transmission for the other eigenbeams causes more loss. That is because zero-forcing scheme is subject to noise enhancement by beam steering. In other words, beam steering applied QRD enables the proposed system to be free from noise enhancement.

IV. CONCLUSION

In this paper, we propose a new space-time turbo equalization scheme that employs beam steering technique based on SVD and QRD is employed in order to support broadband MIMO systems using a single carrier transmission scheme with high spectral efficiency. With this scheme, the computational complexity on a matrix inversion involved in finding LLR can be greatly reduced in the turbo equalizing process. In addition, we have confirmed the proposed scheme is effective in improving spectral efficiency without extra bandwidth with low transmit power.

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